Congruence Closure

# General instruction

## Reduce to flat terms

To terms of the kind

f(x1, … xn )

x = y

x ≠ y

!!! can’t have f(a) ≠ b, must subsitute

Reduction is done via the following transformation (preserving satisfiability)

* Fix a subterm *t*
* Take a new variable *z*
* Replace *t* by *z* everywhere
* Add the further equality *z = t*

## Example

f(x,y) = g(u) ^ f(g(u),u) = x ^ f(b) ≠ c

Becomes

f(x,y) = g(u) ^ f(z,u) = x ^ g(u) = z ^ r ≠ c ^ f(b) = r

## Put terms into equivalent classes (blocks)

If I have t ≠ u I can not write this equality in any box

## Merge blocks

If I have t ≠ u as soon I have t and u in the same block I can stop because it is UNSAT

If t and u are in two different blocks and I can’t do anything more than it is SAT

# Example 1

f(x,y) = g(u) ^ y = u ^ x ≠ g(u) ^ x ≠ y ^ f(x,u) = x ^ f(g(u)) = x

Rewrite g(u) as z, and add the new equality

f(x,y) = z ^ y = u ^ x ≠ z ^ x ≠ y ^ f(x,u) = x ^ f(z) = x ^ g(u) = z

Add the equality in separate blocks

Block 1:

f(x,y)

z

Block 2:

y

u

Block 3:

f(x,u)

x

Block 4:

f(z)

x

Block 5:

g(u)

z

Since f(x,u) and f(z) are both equal to x I can merge blocks 3 and 4, the new blocks are

Block 1:

f(x,y)

z

Block 2:

y

u

Block 3:

f(x,u)

x

f(z)

Block 4:

g(u)

z

Since g(u) and f(x,y) are both equal to z I can merge blocks 1 and 4, the new blocks are

Block 1:

f(x,y)

z

g(u)

Block 2:

y

u

Block 3:

f(x,u)

x

f(z)

Since y = u than f(x,y) = f(x,u), so I can merge blocks 1 and 3, the new blocks are

Block 1:

f(x,y)

z

g(u)

f(x,u)

x

f(z)

Block 2:

y

u

We know that x ≠ z ^ x ≠ y, but x and z are on the same block, so it is UNSAT

# Example 2 (further\_example)

f(g(b)) = a ^ b = c ^ g(c) = a ^ f(a) ≠ a

Rewrite g(b) as z and add the new equation

f(z) = a ^ b = c ^ g(c) = a ^ f(a) ≠ a ^ g(b)= z

!!! can’t have the disequality f(a) ≠ a, substitute f(a) with e and add the new equality

f(z) = a ^ b = c ^ g(c) = a ^ e ≠ a ^ g(b)= z ^ f(a) = e

Create blocks

Block 1:

f(z)

a

Block 2:

b

c

Block 3:

g(c)

a

Block 4:

g(b)

z

Block 5:

f(a)

e

Since blocks 1 and 3 have both a, they can be merged together, the new blocks are

Block 1:

f(z)

g(c)

a

Block 2:

b

c

Block 3:

g(b)

z

Block 4:

f(a)

e

Since b = c (from block 2) than g(b) = g(c), so I can merge block 1 and 3, the new blocks are

Block 1:

f(z)

g(c)

g(b)

z

a

Block 2:

b

c

Block 3:

f(a)

e

Since a = z (from block 1), than f(a) = f(z), so I can merge block 3 and block 1, the new blocks are

Block 1:

f(z)

g(c)

g(b)

z

a

f(a)

e

Block 2:

b

c

Since e ≠ a than e and a must be in two different blocks, but e is in block 1 and a is in block 1 than it is UNSAT